Ex 18

$$\begin{bmatrix} 4 & 5 \\ 6 & 3 \end{bmatrix}$$

$4 \cdot 3 - 5 \cdot 6$

$12 - 30 = -18 \bmod 7$

$= 3$

The inverse of $3 = (3 \cdot \underline{5}) \bmod 7 = 1$

$$\begin{bmatrix} 3 \cdot 5 & -5 \cdot 5 \\ -6 \cdot 5 & 4 \cdot 5 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 5 & 6 \end{bmatrix}$$

$15 \bmod 7 = 1$

$-25 \bmod 7 = 3$

$-30 \bmod 7 = 5$

$20 \bmod 7 = 6$

$$\begin{bmatrix} 4 & 5 \\ 6 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$4 \cdot 1 + 5 \cdot 5 = 29 \bmod 7 = 1$

$4 \cdot 3 + 5 \cdot 6 = 42 \bmod 7 = 0$

$6 \cdot 1 + 3 \cdot 5 = 21 \bmod 7 = 0$

$6 \cdot 3 + 3 \cdot 6 = 36 \bmod 7 = 1$

Scanned with CamScanner

(8)

	5	15	25	35
5	25	35	5	15
15	35	25	15	5
25	5	15	25	35
35	15	5	35	25

- closed $\forall a,b \in S$, $a \cdot b \in S$
- commutative $\forall a,b \in S$, $(a \cdot b) \cdot c = a(b \cdot c) \in S$
- identity $= 25 \in S$
- Inverse

5	15	25	35
5	15	25	35

$U_8 = \{1, 3, 5, 7\} \cdot 5 = 5, 15, 25, 35$

(15)

$n=1$ is true $(ab)^1 = a^1b^1$

$$ab = ab$$

assume that $n=k$ is true

$$(ab)^k = a^k b^k$$

$n=k+1$

$$\begin{aligned}(ab)^{k+1} &= (ab)(ab)^k \\ &= (ab)(a^k b^k) \\ &= a(ba^k)b^k \\ &= a(a^k b)b^k \\ &= (aa^k)(bb^k) \\ &= a^{k+1}b^{k+1}\end{aligned}$$